

Analysis of a Lens Based Antenna Using Spherical Modes

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Abstract - A fixed shaped beam antenna offers an excellent compromise between cost and system performance in high data rate systems operating in the frequency range of 5- 65 GHz. Shaped Dielectric Lenses perform the task of collimation and shaping, along with physically small feed antennas, to obtain multiple and shaped beams in a fixed set of directions. Lenses are inherently broadband, easy to fabricate, have lower dimensional tolerances, cost effective and act as radome for the primary radiators that are embedded inside or placed behind the lens. Earlier techniques for the analysis of shaped dielectric lenses, employed ray tracing methods of GO and PO, that are valid only in the far field of a primary point source type of radiator. But the dielectric lens is in the near field of finite sized primary radiators oriented at different angles and at different distances from the lens center. In this paper we discuss, a new accurate analytical procedure for the radiation pattern of a spherical lens excited by a rectangular patch. The lens is treated as a scatterer. Techniques for radial translation and spatial rotation of the small aperture Spherical Modal Complex Coefficients (SMCC) are utilized to align them to the phase center of the Dielectric Lens. The SMCC of the scattered fields due to the lens are then obtained by application of boundary conditions. Sample computations are performed to demonstrate the approach and supported by experimental results.

Index Terms - MBS, Smart Antenna, Dielectric Lens, SME, SVWF, SMCC

1. INTRODUCTION

New frequency bands were opened up in 5 to 11 GHz, to support WLAN applications and in 40 to 65 GHz bands to support Mobile Broadband Systems (MBS). To achieve a high rate goal, 'Smart Antennas' were proposed at the base stations[1]. An adaptive multi beam smart antenna has the ability to cope with a rapidly changing environment by distributing the limited power of solid state amplifiers in the best possible way within the cell, as to ensure sufficient signal to noise ratio at a receiver to sustain the desired high data rates. However a cost effective implementation of a 'Smart Antenna' is limited due to high cost of RF array elements, RF distribution and phasing [1-2].

It is for this reason that fixed shaped beam antennas offer an excellent compromise between cost and system performance especially in an urban scenario where dense traffic occurs in specified directions only.

Shaped Dielectric Lenses perform the task of collimation and shaping, along with physically small feed antennas, to obtain multiple and shaped beams in a fixed set of directions. Lenses are inherently broadband, easy to fabricate, have lower dimensional tolerances, cost effective and provide a covering radome for the primary radiators that are embedded inside or placed behind the lens. The drawback thus far has primarily been an accurate analysis, apart from reducing internal reflections, dielectric losses and eventually the size of the dielectric lens [2-4].

Earlier techniques for the analysis of shaped dielectric lenses, employed ray tracing methods of GO and PO, that are valid only in the far field of a primary point source type of radiator. But in reality, the dielectric lens is in the near field of finite sized primary radiators oriented at different angles and at different distances from the lens center.

In this work, a new accurate analytical procedure is proposed for the radiation pattern of multiple primary radiators in the near field of a dielectric lens using Spherical

Modal Expansion (SME) approach. Earlier SME approaches to solve for the field scattered by a dielectric sphere approximated the primary radiator as a far field Huygens source [9].

The lens is treated as a scatterer. Techniques for radial translation and spatial rotation of the small aperture Spherical Modal Complex Coefficients (SMCC) are utilized to align them to the phase center of the Dielectric Lens. The SMCC of the scattered fields due to the lens are then obtained by application of boundary conditions.

The analysis is flexible enough to accommodate different types of radiators and different shapes for the dielectric lens.

2. ANALYSIS

Consider a set of primary feeds radiating in the presence of a dielectric sphere as shown below.

The co-ordinate system of the lens in which the primary feeds are embedded is shown in Figure1a [9]. Figure1b is the coordinate system native to a primary feed.

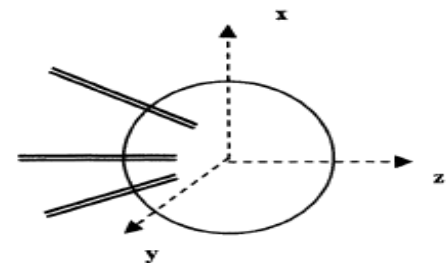


Figure 1a A Lens with three feeds

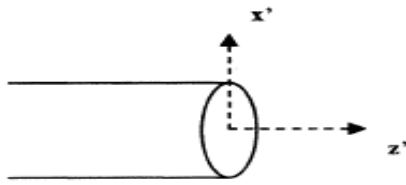


Figure 1b A single primary feed

A. Characterization of the Primary Radiator

The near fields radiated by the primary feed at any point in its own coordinate system are given by

$$E_i(R', \theta', \phi') = \sum_n (a_n M'_{mn} + b_n N'_{mn}) \quad (1)$$

$$H_i(R', \theta', \phi') = (-jY_0) \sum_n (a_n N'_{mn} + b_n M'_{mn}) \quad (2)$$

Here M'_{mn} and N'_{mn} are the mutually orthogonal TE & TM Spherical Vector Wave Functions (SVWF) respectively, defined in [10] with a $e^{j\omega t}$ time variation. The TE and TM Spherical Modal Complex Coefficients (SMCC) are denoted by a_n and b_n respectively. Here 'n' is the spherical modal index and 'm' is the azimuthal index occurring as $e^{jm\phi}$. For linearly polarized primary radiators $m = \pm 1$. Y_0 is the admittance of the surrounding medium. The maximum value of the running index 'n' is set to $N_{\max} = \text{ceil}[k * \text{lensdia}]$ that accounts for more than 99.9% of the energy radiated by collimating type of antennas as shown by Arthur [12] and like the ones considered in this work.

Compute the far fields over an enclosing sphere centered at the aperture center by aperture field integration described in [11]. The fields over an enclosing sphere may also be obtained by an accurate measurement for any one primary radiator in an anechoic chamber. Using the orthogonality of the SVWF's, the SMCC's a_n and b_n of the primary radiator are obtained. These are valid anywhere in near and far space [9].

B. Radial Translation of SMCC

The SMCC of the primary radiator are to be translated to the phase center of the dielectric sphere, as it is the reference point for evaluating the fields finally scattered by the lens. This is accomplished by translation addition theorems and a recursion method of computation described in [6-7]. But it should be noted that while the translated origin would coincide with the phase center and coordinate origin of the dielectric lens, it is not yet spatially aligned to it. The translated SMCC would have the following expressions.

$$E_i(R'', \theta'', \phi'') = \sum_n (A_{tn} M''_{mn} + B_{tn} N''_{mn}) \quad (3)$$

$$H_i(R'', \theta'', \phi'') = (-jY_0) \sum_n (A_{tn} N''_{mn} + B_{tn} M''_{mn}) \quad (4)$$

In equation(3) & (4) the symbol '' is used to represent a radiator coordinate system that has been translated to the lens phase center but not spatially aligned to it. The translated SMCC's A_{tn} and B_{tn} are computed for a distance 'd' between the primary radiator aperture centre and the lens phase centre (Figure 1a) using the expressions below.

$$A_{tn} = \sum_v (a_n A_{vn} + b_n B_{vn}) \quad (5)$$

$$B_{tn} = \sum_v (a_n B_{vn} + b_n A_{vn}) \quad (6)$$

The running index 'v' is up to N_{\max} . The translation coefficients A_{vn} and B_{vn} in (5) and (6) are given by [6].

$$A_{vn} = C(v, n) \sum_p [(-j)^p \{n(n+1) + v(v+1) * a(m, n, -m, v, p)\} Z_p(kd)] \quad (7)$$

$$B_{vn} = -C(v, n) \sum_p [(-j)^p \{2jmkd * a(m, n, -m, v, p)\} Z_p(kd)] \quad (8)$$

Where

$$C(v, n) = (-1)^{2m+v} (-j)^{v-n} \frac{(2v+1)}{2v(v+1)} \quad \text{and}$$

$Z_p(kd) = h_n^2(kd)$ is the spherical Hankel function of second kind.

The summation over 'p' runs as $|n-v|, \dots, n+v$ and includes $1 + \max[v, n]$ terms. The values of $a_p = a(m, n, -m, v, p)$ are computed using the recursion relations [6] as below.

$$\alpha_{p-3} \alpha_{p-4} - (\alpha_{p-2} + \alpha_{p-1} + 4m^2) \alpha_{p-2} + \alpha_p \alpha_p = 0 \quad (9)$$

Where

$$a_p = a(m, n, -m, v, p), \quad p = n+v, n+v-2, \dots, (n-v) \quad \text{and}$$

$$\alpha_p = \frac{[(n+v+1)^2 - p^2][p^2 - (n-v)^2]}{4p^2 - 1} \quad (10)$$

The recursion relation is performed in the descending order of the index.

C. Spatial Rotation of SMCC

The translated SMCC in equations (5) and (6) are now referred to the lens phase centre coordinate system by performing a spatial rotation. If R represents the mathematical rotation group, then as described by

Edmonds [8], the symbol $D_{\mu}^n(R)$ would be the matrix of

rotation coefficients for one element of the rotation group defined by a set of three Eulerian angles (ϕ, θ, ψ) that would align the translated primary radiator coordinate system with the lens phase centre coordinate system. The translated and rotated SMCC of the primary radiator are computed using A_{tm} and B_{tm} in (5) and (6) after substituting into the expressions below.

$$A_{nm} = \sum_u A_{tm} D_{mu}^n(R) \quad (11)$$

$$B_{nm} = \sum_u B_{tm} D_{mu}^n(R) \quad (12)$$

$$D_{mu}^n(R) = (e^{-jm\phi}) * d_{mu}^n(\theta) * (e^{-jn\psi}) \quad (13)$$

In (11), (12) and (13), 'u' is the polarization index and 'm' is the azimuthal index. In our case the primary radiators chosen always have $u = m = 1$. The rotation factors $d_{mu}^n(\theta)$ are computed from the recursion relations for Jacobi polynomials for small values of 'n' as described in [8]. For large values of 'n' it is computationally more efficient to compute the rotation factors by employing complex FFT and a data reduction techniques. Since we are dealing with N_{max} of less than 50 the recursion relation was preferred here.

D. Scattering Coefficients

Boundary conditions are applied on the surface of the dielectric lens for the total field, which are represented as $E_i + E_s = E_d$ and $H_i + H_s = H_d$. Here E_i is the incident field of the primary radiator, E_s is the field scattered outside the lens and E_d is the field scattered inside the lens. The incident field E_i is now characterized by its Translated and Rotated SMCC $[A_{nm}, B_{nm}]$ as in (11) and (12) referred to the lens phase centre. The unknown scattered fields E_s and E_d are represented by their unknown SMCC $[A_{ns}, B_{ns}]$ and $[A_{nd}, B_{nd}]$ respectively now referred to unprimed lens phase center coordinate system. Substituting the spherical modal expressions for E_i, E_s and E_d in terms of their SMCC, it is evident that there are two known coefficients $[A_{nm}, B_{nm}]$ and four unknown coefficients $[A_{ns}, B_{ns}], [A_{nd}, B_{nd}]$ for each modal index 'n'. Thus there are $4 * N_{max}$ spherical modal complex coefficients that are unknown and to be determined. To solve for these, we apply equations of continuity of tangential electric and magnetic field $\bar{E} = E_\theta \bar{\theta} + E_\phi \bar{\phi}$ over the surface of the sphere $r = r_0$

$$E_{i\theta} + E_{s\theta} = E_{d\theta} \quad (14)$$

$$E_{i\phi} + E_{s\phi} = E_{d\phi} \quad (15)$$

$$H_{i\theta} + H_{s\theta} = H_{d\theta} \quad (16)$$

$$H_{i\phi} + H_{s\phi} = H_{d\phi} \quad (17)$$

In the case of a spherical dielectric lens of radius r_0 , the orthogonality conditions of SVWF [10] can be invoked on the surface of the sphere by integrating the dot product of \bar{E} with the vector wave functions M_{mn} and N_{mn} over ϕ and applying orthogonality of the Associated Legendre functions in θ , to obtain four linearly independent equations for each spherical modal index 'n'. Thus all the $4 * N_{max}$ unknowns are easily solved as closed form expressions and given below for a spherical lens.

$$A_{nd} = A_{nm} [\delta j_n(kr_0) h_n^2(kr_0) - j_n(kr_0) \delta h_n^2(kr_0)] / W_1 \quad (18)$$

$$A_{ns} = A_{nm} [\tilde{\delta} j_n(kr_0) j_n(k_d r_0) - (Y_d / Y_0) \tilde{\delta} j_n(k_d r_0) j_n(kr_0)] / W_1 \quad (19)$$

$$B_{nd} = B_{nm} [j_n(kr_0) \delta h_n^2(kr_0) - \delta j_n(kr_0) h_n^2(kr_0)] / W_2 \quad (20)$$

$$B_{ns} = B_{nm} [\tilde{\delta} j_n(k_d r_0) j_n(kr_0) - (Y_d / Y_0) j_n(k_d r_0) \tilde{\delta} j_n(kr_0)] / W_2 \quad (21)$$

Here δ is the differential operator expanded typically as

$$\delta j_n = \frac{1}{kr} \frac{d[r j_n(kr)]}{dr} \quad \text{and}$$

$$W_1 = \left[\left(\frac{y_d}{y_o} \right) \delta j_n(k_d r_0) h_n^2(kr_0) - j_n(k_d r_0) \delta h_n^2(kr_0) \right] \quad (22)$$

$$W_2 = \left[\left(\frac{y_d}{y_o} \right) j_n(k_d r_0) \delta h_n^2(kr_0) - \tilde{\delta} j_n(k_d r_0) h_n^2(kr_0) \right] \quad (23)$$

When the lens shape is not spherical, a closed form solution may not be possible. However this technique can be easily extended to dielectric shells easily. The scattered field outside the shell is obtained by the same method but the field inside the spherical shell needs reapplication of the boundary conditions that yields a different set of SMCCs inside the spherical shell.

With all SMCC for E_i as $[A_{nm}, B_{nm}]$ and E_s as $[A_{ns}, B_{ns}]$ now known, the total field at any point in space in the near or far field is computed by addition of the incident and scattered fields using the equations (1) and (2).

3. SAMPLE CALCULATION AND EXPERIMENT

A sample computation was performed to test the validity of the analysis. A single element rectangular patch was made to radiate at a frequency of 2.5 GHz, in front of a Teflon sphere ($\epsilon_d = 2.08$) of diameter 12cms in an anechoic chamber. The distance of separation between the patch and center of the dielectric lens was kept at 5 cm, where the reflection losses were better than -20 db. To perform the measurement, the dielectric sphere was kept on a supporting stand made of polystyrene foam with $\epsilon_d = 1.08$ practically same as free space, so that the radiation pattern of the aperture-lens combination does not get perturbed.

Figure 2a and 2b show the single element rectangular patch and the spherical lens fabricated for the purpose respectively. Figure 3 shows the actual set up used for the experiment. Figures 4 show the return losses of the patch antenna.

The rectangular patch-lens combination was excited by a RF signal generator at a frequency of 2.7GHz. At this frequency, the number of modes considered was, $N_{\max} \cong 30$ i.e. $N_{\max} = \text{ceil}[k * \text{lensdia}]$ which accounts for about 99.9% of the energy.

The SMCCs a_n and b_n of the primary radiator are computed using the field expressions (1) and (2) of the SME model by exploiting the orthogonality properties of M_{nm} and N_{nm} [11]. The magnitudes of a_n and b_n are shown in Figure 5 for 100 modes. The fast decay of the SMCCs indicates that most of the energy is concentrated in the first few modes (~5) whereas the higher order modes contain negligible energy.

Using these SMCCs, the far fields of the rectangular patch are reconstructed by substituting them in expressions (1) and (2). The fields of the rectangular patch were also calculated using Aperture Integration Method [11]. There was a satisfactory match between the patterns obtained by the two methods as is evident from Figure 6.

As the phase centre of the dielectric lens is the reference point for the analysis, the SMCC's obtained in the previous steps are subjected to the operation of radial translation using the equations (5) and (6) for a radial distance of 2.5cms.

As expected, the SMCCs decay fast indicating that the translation operation has not changed the energy content of the modes. The far fields are reconstructed again using the SME expressions (1) and (2), but with the translated SMCCs and are shown in Figure 7. The translated SMCCs are then spatially rotated using the rotation matrix to align them to the phase centre of the lens with the help of expressions in (11) and (12). The fields after the rotation operation are calculated by substituting the rotated SMCCs in expressions (1) and (2) and is shown in Figure 8. There is a reasonably close match between these fields and those in Figure 6.

The scattering coefficients A_{nd} , A_{ns} , B_{nd} , and B_{ns} , are obtained by invoking the boundary conditions on the lens surface for the total fields using the expressions (18)-(21). The fields E_s and E_d scattered by the lens are found out by substituting these scattering coefficients in (1) and (2). Figure 9 shows the fields scattered by the lens, which are in close agreement with the fields obtained by direct aperture integration shown in Figure 6.



Figure 2a Rectangular Patch



Figure 2b Teflon Spherical Lens

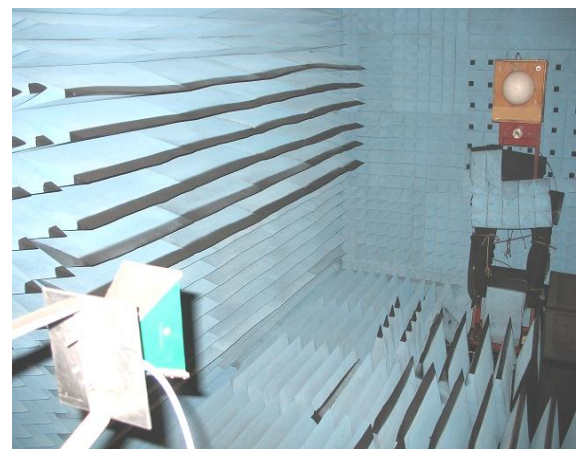


Figure 3 Lens Setup in an Anechoic Chamber

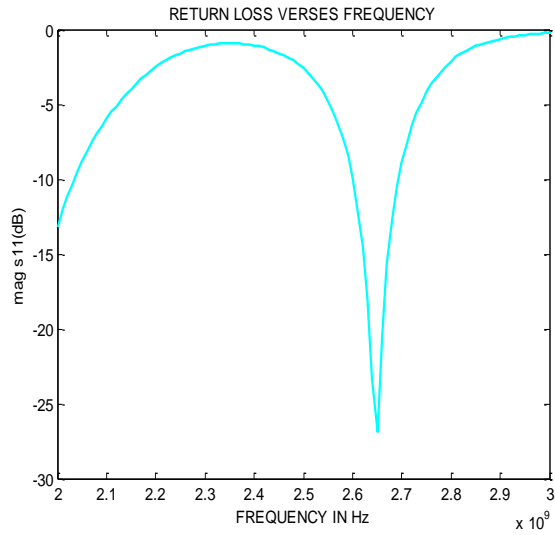


Figure 4 Return Loss of the Patch Antenna

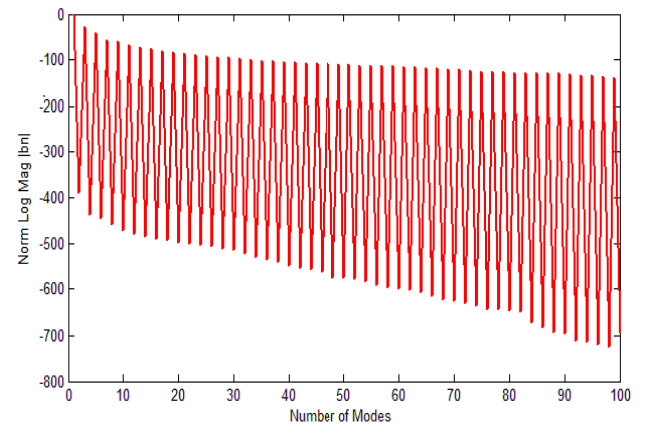


Figure 5 Plot of a_n and b_n

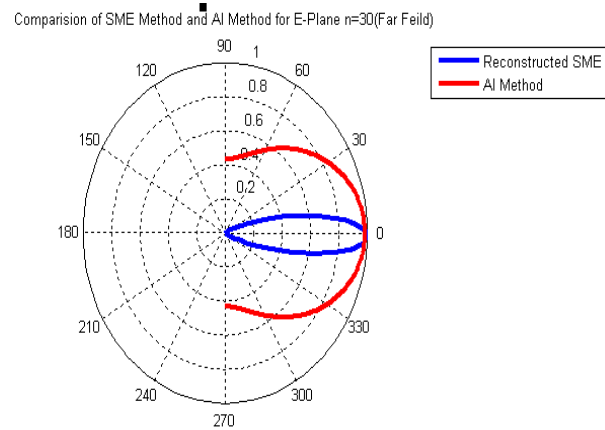
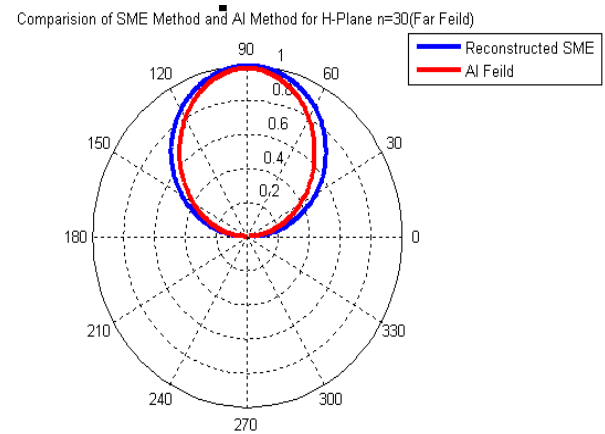
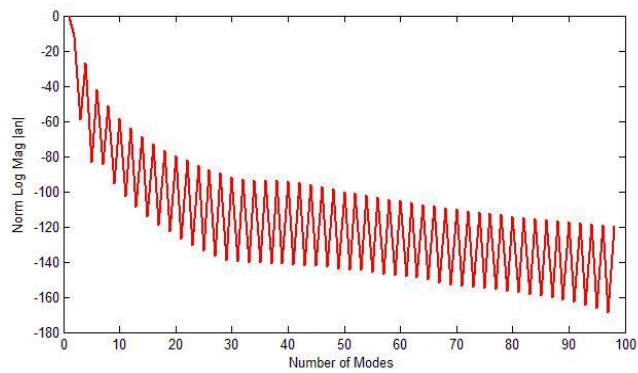


Figure 6 Far Fields calculated by SME and AI Methods (Initial Reconstruction)

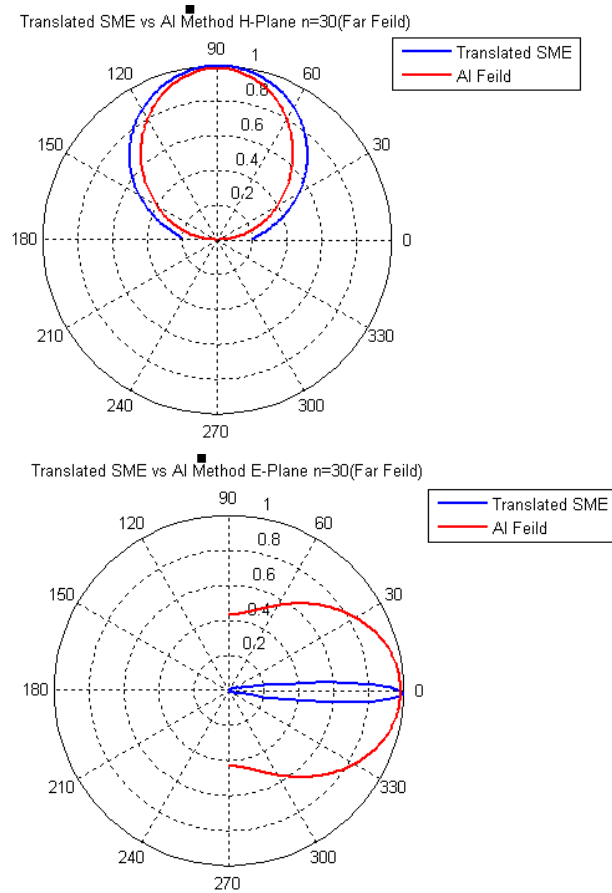


Figure7 Far Fields after linear translation

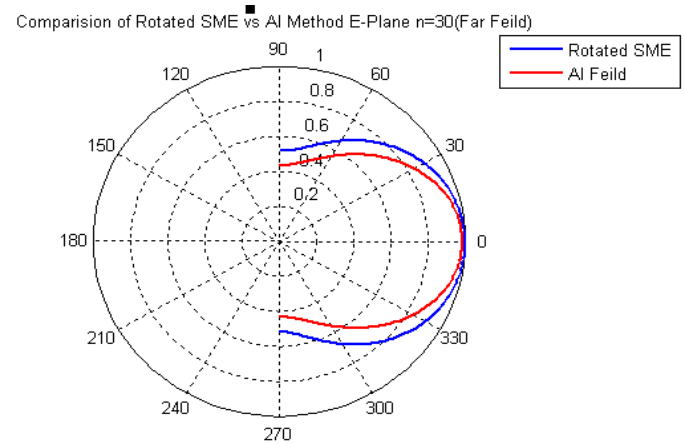


Figure 8 Far Fields after spatial rotation

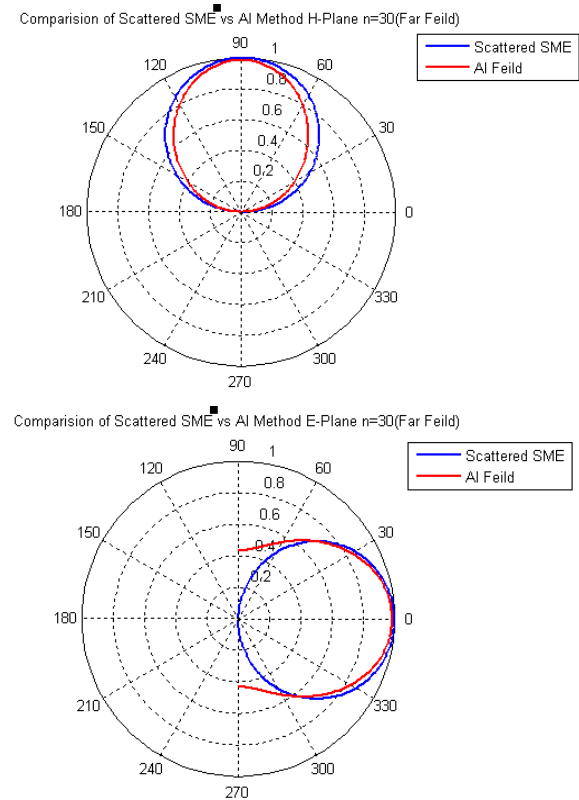


Figure 9 Far Fields after scattering

4. CONCLUSION

An analytically accurate and novel solution to the problem of prediction of radiation patterns of small aperture primary radiators in the presence of dielectric lenses has been presented. This uses Spherical Modal Expansion to characterize the primary radiator fields and the unknown scattered fields are obtained from a closed form solution for

spherically shaped lenses. The procedure involved translation and rotation of the primary radiator SME coefficients. The validity of this theoretical procedure has been demonstrated for a typical case of primary radiators like rectangular waveguide, small horns and patch arrays radiating in the presence of a Teflon spherical lens by comparing results with an experimental program. The analysis is easily applied to any type of radiating aperture and any shaped dielectric lens.

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ACKNOWLEDGEMENTS

The authors sincerely acknowledge the financial support by ISRO (Indian Space Research Organization), Bangalore, India.